Analysis of the Time Response of Multiconductor Transmission Lines with Frequency-Dependent Losses by the Method of Convolution-Characteristics

Jun-Fa Mao and Zheng-Fan Li

Abstract—**A new method for analysis of the time response of multiconductor transmission lines with frequency-dependent losses is presented. This method can solve the time response of various kinds of transmission lines with arbitrary terminal networks. Particularly it can analyze nonuniform lines with frequency-dependent losses, for which there is no existing effective method to analyze their time response so far. This method starts from the frequency-domain telegrapher's equations. After decoupling and inversely Fourier transforming, then a set of decoupled time-domain equations including convolutions are given. These equations can be solved with the characteristic method. The results obtained with this method are stable and accurate. Two examples are given to illustrate the application of this method to various multiconductor transmission lines.**

I. Introduction

In LARGE-SCALE high-speed integrated circuits, multiconductor transmission lines are usually used as interconnections. When the speed is relatively high, the delay, crosstalk, and distortion of signals [1], [2] are obvious. The time response of these lines has been of interest for many years and well studied [3]–[9]. For different kinds of lines, there are different methods for analysis.

For uniform lines, lossless lines can be easily analyzed by the modal analysis method in the time-domain [10]. Arbitrarily loaded lossy uniform lines with frequency-independent parameters have been analyzed by the characteristic method [6] *et al.* Linearly loaded uniform lines with frequency-dependent parameters can be studied with the frequency-domain method [7] *et al.* For nonlinearly loaded uniform lines with frequency-dependent parameters, there have been several efficient methods to analyze [11]–[13]. In these methods the relations between the response signals at the input and output ends of the lines are first obtained in the frequency-domain by solving the frequency-domain telegrapher's equations directly or indirectly, then transformed into the time-domain either by convolution [11], [13] or by polynomial ratio approximation [12].

About the nonuniform lines, several methods have been delivered for analysis in the case of frequency-independent parameters [8], [14], [15], but these methods are difficult to be applied to the case of frequency-dependent parameters. If nonuniform lines with frequency-dependent parameters are approximated by many short sections of uniform lines, the methods as given in [11]–[13] may be able to analyze them, but this would make the problem complicated. So there is no efficient method for analysis of the time response of nonuniform lines with frequency-dependent parameters so far.

In this paper, a method of convolution-characteristics is presented. Convolution is a usual technique to deal with the frequency-dependence of line parameters, and the characteristic method has been applied to analyze nonuniform lines with frequency-independent parameters [14]. The method of convolution-characteristics in this paper combines the functions of both convolution and characteristics. This method can analyze the time response of nonuniform lines with frequency-dependent losses under arbitrary load conditions. Of course, as a general method, it can also analyze the various kinds of uniform transmission lines described above. It starts from the telegrapher's equations in the frequency-domain. The structure and coupling mode of the transmission lines considered may be arbitrary, but for the reason of illustrating this method more clearly, two assumptions are made in this paper. Under these assumptions, the telegrapher's equations in the frequency-domain are easily decoupled and then inversely Fourier transformed. Thus a set of decoupled time-domain equations including convolutions are gotten, the convolutions are produced due to the frequency-dependent losses. These decoupled equations can be numerically solved using the characteristic method [16] in which the segmentation of each decoupled line is taken, and this is why the nonuniformity of the lines can be studied. Two examples are given to illustrate the application of this method, and some results are favorably compared with those obtained with other methods.

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II. Theory

In this paper multiconductor transmission lines are considered to be quasi-TEM lines so they satisfy the telegrapher's equations. In general case, the parameters of the transmission lines are frequency-dependent considering the ultra-broadband spectrum occupied by the high-speed signals, but only the frequency-dependent performance of resistances and conductances of lines is obvious. Capacitances are almost frequency-independent. For inductances of lines, their internal inductances vary with frequency, so the total inductances are frequency-dependent [4], but the variation of them is unremarkable because the internal inductances are only small parts of the total inductances for general transmission system structure in large-scale high-speed integrated circuits. Hence in practical case, only the frequency-dependence of resistances and conductances of the lines must be considered, while the inductances and capacitances are approximated to be independent of frequency.

As will be seen, the frequency-dependence of conductances can be dealt with the same way to that of resistances. On the other hand, conductances have less influence on the time response of the lines than resistances in practical transmission lines, so in this paper the conductances are also considered to be frequency-independent for simplification. Then the telegrapher's equations of the multiconductor lines in the frequency-domain can be written as

$$\partial [V(x, \omega)]/\partial x = -[R(x, \omega)]\,[I(x, \omega)] - jw[L(x)]\,[I(x, \omega)] \tag{1}$$

$$\partial [I(x, \omega)]/\partial x = -[G(x)]\,[V(x, \omega)] - jw[C(x)]\,[V(x, \omega)] \tag{2}$$

where $[L(x)]$, $[C(x)]$, $[R(x, \omega)]$ and $[G(x)]$ are respectively inductance, capacitance, resistance and conductance N-by-N (N is the number of the lines) matrices per unit length. They are all functions of position (x) on the lines in general nonuniform case, but from the above discussions, only $[R(x, \omega)]$ is still function of angular frequency (w) because of skin effect and proximity effect.

Equations (1) and (2) are coupled ones, there are several methods to decouple them [9], [14], [17]. Here we decouple equations (1) and (2) with the method in [17] to illustrate our method in this paper more clearly. In the method in [17], two assumptions on the structure of the coupled lines are made. One is that each transmission line is coupled directly only with the closest one to the left and with the closest one to the right. This assumption is valid in some practical cases, in particular when the transmission lines are microstrips on a multilayer printed circuit board where every signal plane is sandwiched between two ac ground planes. Another assumption is that the lines are identical and equally spaced and side effects are negligible. This assumption is reasonable for some transmission lines, too. In [18] the errors introduced by these two assumptions are analyzed and a method to find the upper bounds on such errors is developed. Under these assumptions $[L(x)]$, $[C(x)]$, $[R(x, \omega)]$ and $[G(x)]$ are all symmetric tridiagonal Toeplitz matrices at any position (x) and frequency (w). Matrices of this kind have an important property in which they can be diagonalized with a matrix $[M]$ which is the matrix of right eigenvectors of such matrices and is decided only by N, no matter what value of the elements of such matrices are. Let

$$[V(x, \omega)] = [M]\,[\overline{V}(x, \omega)] \tag{3}$$

$$[I(x, \omega)] = [M]\,[\overline{I}(x, \omega)] \tag{4}$$

and insert them into (1) and (2), a set of decoupled equations are then obtained:

$$\partial [\overline{V}(x, \omega)]/\partial x = -[\overline{R}(x, \omega)]\,[\overline{I}(x, \omega)] - jw[\overline{L}(x)]\,[\overline{I}(x, \omega)] \tag{5}$$

$$\partial [\overline{I}(x, \omega)]/\partial x = -[\overline{G}(x)]\,[\overline{V}(x, \omega)] - jw[\overline{C}(x)]\,[\overline{V}(x, \omega)] \tag{6}$$

where $[\overline{L}(x)] = [M]^{-1}[L(x)]\,[M]$ is diagonal, so are $[\overline{C}(x)]$, $[\overline{R}(x, \omega)]$ and $[\overline{G}(x)]$. $[M]$ is the matrix of right eigenvectors of N-by-N Toeplitz matrices.

Note that the above two assumptions are not restrictions of the method in this paper. For the general transmission lines with arbitrary structure, the decoupling method in [14] applies. In that method in [14], $[L(x)]$ and $[C(x)]$ are diagonalized by the matrices of eigenvectors of $[L(x)]\,[C(x)]$ and $[C(x)]\,[L(x)]$. As long as $[L(x)]$ and $[C(x)]$ are diagonalized, the characteristic method can be applied, no matter whether $[R(x, \omega)]$ and $[G(x)]$ are diagonalized.

Equations (5) and (6) can be rewritten as

$$\partial \overline{V}_i(x, w)/\partial x = -\overline{R}_i(x, w)\overline{I}_i(x, w) - jw\overline{L}_i(x)\overline{I}_i(x, w) \tag{7}$$

$$\partial \overline{I}_i(x, w)/\partial x = -\overline{G}_i(x)\overline{V}_i(x, w) - jw\overline{C}_i(x)\overline{V}_i(x, w) \qquad i = 1, 2, \cdots, N \tag{8}$$

where $\overline{L}_i(x)$, $\overline{C}_i(X)$, $\overline{R}_i(x, w)$ and $\overline{G}_i(x)$ are the ith diagonal elements of matrices $[L(x)]$, $[C(x)]$, $[R(x, \omega)]$ and $[G(x)]$, respectively, $\overline{V}_i(x, w)$ and $\overline{I}_i(x, w)$ are the ith elements of vectors $[\overline{V}(x, \omega)]$ and $[\overline{I}(x, \omega)]$.

Now inversely Fourier transform (7) and (8), one can get

$$\partial \overline{V}_i(x, t)/\partial x = T_i(x, t) - \overline{L}_i(x)\partial \overline{I}_i(x, t)/\partial t \tag{9}$$

$$\partial \overline{I}_i(x, t)/\partial x = -\overline{G}_i(x)\overline{V}_i(x, t) - \overline{C}_i(x)\partial \overline{V}_i(x, t)/\partial t \tag{10}$$

in which $\overline{V}_i(x, t)$ and $\overline{I}_i(x, t)$ are the inverse Fourier transforms of $\overline{V}_i(x, w)$ and $\overline{I}_i(x, w)$, $i = 1, 2, \cdots, N$. If $[V(x, t)]$ and $[I(x, t)]$ are the time response we want, and $[\overline{V}(x, t)]$ and $[\overline{I}(x, t)]$ are the vectors whose elements are $\overline{V}_i(x, t)$ and $\overline{I}_i(x, t)$, because $[M]$ is a constant matrix, it is easy to see that

$$[V(x, t)] = [M]\,[\overline{V}(x, t)] \qquad (11)$$

$$[I(x, t)] = [M]\,[\overline{I}(x, t)]. \qquad (12)$$

In (9), $T_i(x, t)$ is a convolution:

$$T_i(x, t) = \int_0^t \overline{R}_i(x, \tau)\overline{I}_i(x, t - \tau)\, d\tau \qquad (13)$$

where $\overline{R}_i(x, t)$ is the inverse Fourier transform of $\overline{R}_i(x, w)$. $T_i(x, t)$ can be numerically calculated which will be seen below.

Equations (9) and (10) are decoupled ones in the time-domain and can be solved with the generalized characteristic method. Let

$$dx/dt = 1/\sqrt{\overline{L}_i(x)\overline{C}_i(x)}, \qquad i = 1, 2, \cdots, N$$

from (9) and (10), one can get

$$d(\overline{V}_i + \sqrt{\overline{L}_i/\overline{C}_i}\overline{I}_i)/dt = -(\overline{V}_i\overline{G}_i/\overline{C}_i + T_i/\sqrt{\overline{L}_i\overline{C}_i}) + \overline{I}_i FD_i \qquad i = 1, 2, \cdots, N \qquad (14)$$

where brackets have been omitted for brevity, and

$$FD_i = FD_i(x) = \frac{1}{2\overline{L}_i(x)}\, d\left(\frac{\overline{L}_i(x)}{\overline{C}_i(x)}\right)\bigg/ dx$$

Similarily, let $dx/dt = -1/\sqrt{\overline{L}_i(x)\overline{C}_i(x)}$, there is

$$d(\overline{V}_i - \sqrt{\overline{L}_i/\overline{C}_i}\overline{I}_i)/dt = (\overline{V}_i\overline{G}_i/\overline{C}_i - T_i/\sqrt{\overline{L}_i\overline{C}_i}) + \overline{I}_i FD_i \qquad i = 1, 2, \cdots, N. \qquad (15)$$

Equations (14) and (15) can be given their numerical forms with the trapezoid algorithm:

$$\begin{aligned}
&\overline{V}_i(x_{k+1}, t_{n+1}) + \sqrt{\overline{L}_i(x_{k+1})/\overline{C}_i(x_{k+1})}\,\overline{I}_i(x_{k+1}, t_{n+1}) \\
&= \overline{V}_i(x_k, t_n) + \sqrt{\overline{L}_i(x_k)/\overline{C}_i(x_k)}\,\overline{I}_i(x_k, t_n) \\
&\quad + .5\Delta t[-\overline{V}_i(x_k, t_n)\overline{G}_i(x_k)/\overline{C}_i(x_k) - T_i(x_k, t_n)/ \\
&\quad \sqrt{\overline{L}_i(x_k)\overline{C}_i(x_k)} - \overline{V}_i(x_{k+1}, t_{n+1})\overline{G}_i(x_{k+1})/ \\
&\quad \overline{C}_i(x_{k+1}) + \overline{I}_i(x_k, t_n)FD_i(x_k) - T_i(x_{k+1}, t_{n+1})/ \\
&\quad \sqrt{\overline{L}_i(x_{k+1})\overline{C}_i(x_{k+1})} \\
&\quad + \overline{I}_i(x_{k+1}, t_{n+1})FD_i(x_{k+1})] \qquad (16)
\end{aligned}$$

$$\begin{aligned}
&\overline{V}_i(x_{k-1}, t_{n+1}) - \sqrt{\overline{L}_i(x_{k-1})/\overline{C}_i(x_{k-1})}\,\overline{I}_i(x_{k-1}, t_{n+1}) \\
&= \overline{V}_i(x_k, t_n) - \sqrt{\overline{L}_i(x_k)/\overline{C}_i(x_k)}\,\overline{I}_i(x_k, t_n) \\
&\quad + .5\Delta t[-\overline{V}_i(x_k, t_n)\overline{G}_i(x_k)/\overline{C}_i(x_k) + T_i(x_k, t_n)/ \\
&\quad \sqrt{\overline{L}_i(x_k)\overline{C}_i(x_k)} - \overline{V}_i(x_{k-1}, t_{n+1})\overline{G}_i(x_{k-1})/ \\
&\quad \overline{C}_i(x_{k-1}) + \overline{I}_i(x_k, t_n)FD_i(x_k) + T_i(x_{k-1}, t_{n+1})/ \\
&\quad \sqrt{\overline{L}_i(x_{k-1})\overline{C}_i(x_{k-1})} + \overline{I}_i(x_{k-1}, t_{n+1})FD_i(x_{k-1})] \\
&\quad i = 1, 2, \cdots, N \qquad (17)
\end{aligned}$$

where $t_{n+1} = n\Delta t$, Δt is the time step; x_{k-1}, x_k and x_{k+1} are positions of segmenting points on lines which can be obtained [19] as

$$\int_0^{x_k} \sqrt{\overline{L}_i(x)\overline{C}_i(x)}\, dx = (k - 1)\,\Delta t \qquad i = 1, 2, \cdots, N. \qquad (18)$$

From this equation, x_k is easily solved numerically on computer.

It is equivalent to consider that decopling equations (1) and (2) actually separates the coupled transmission line system into N decoupled single lines of parameters $\overline{L}_i(x)$, $\overline{C}_i(x)$, $\overline{R}_i(x, w)$ and $\overline{G}_i(x)$. Each single line has a transmission mode on it with velocity $1/\sqrt{\overline{L}_i(x)\overline{C}_i(x)}$. Equation (18) indicates that different single line is differently segmented, and the total numer (NI_i) of segments of each line is different too:

$$NI_i = \left(\int_0^D \sqrt{\overline{L}_i(x)\overline{C}_i(x)}\, dx\right)\bigg/\Delta t$$

in which D is the length of line.

There is a thing to be mentioned. The item

$$FD_i(x) = \frac{1}{2\overline{L}_i(x)}\, d\left(\frac{\overline{L}_i(x)}{\overline{C}_i(x)}\right)\bigg/ dx$$

requires that the transmission lines must be continual. If the nonuniformity of lines takes the form of discontinuities, the method [16] for interconnected uniform lines applies, which isn't within the interest of this paper.

In (16) and (17), the convolution items are the most complicated and just where the crux of this method lies. As mentioned before they can be calculated numerically. For example

$$\begin{aligned}
T_i(x_k, t_n) &= \sum_{i=0}^{n-2} \overline{R}_i(x_k, j\Delta t)\overline{I}_i(x_k, (n - 1 - j)\,\Delta t)\,\Delta t \\
&= \sum_{i=1}^{n-2} \overline{R}_i(x_k, j\Delta t)\overline{I}_i(x_k, (n - 1 - j)\,\Delta t)\,\Delta t \\
&\quad + \overline{R}_i(x_k, 0)\overline{I}_i(x_k, (n - 1)\,\Delta t)\,\Delta t.
\end{aligned}$$

Considering another expression for $T_i(x_k, t_n)$:

$$T_i(x_k, t_n) = \sum_{j=1}^{n-1} \overline{R}_i(x_k, j\Delta t)\overline{I}_i(x_k, (n - 1 - j)\,\Delta t)\,\Delta t$$

$T_i(x_k, t_n)$ can be more accurately calculated as

$$\begin{aligned}
T_i(x_k, t_n) &= .5\overline{R}_i(x_k, 0)\overline{I}_i(x_k, (n - 1)\,\Delta t)\,\Delta t + TT_i(x_k, t_n) \\
&\quad + .5\overline{R}_i(x_k, (n - 1)\,\Delta t)\overline{I}_i(x_k, 0)\,\Delta t \qquad (19)
\end{aligned}$$

with

$$TT_i(x_k, t_n) = \sum_{j=1}^{n-2} \overline{R}_i(x_k, j\Delta t)\overline{I}_i(x_k, (n - 1 - j)\,\Delta t)\,\Delta t \qquad (20)$$

Similarly for $T_i(x_{k\pm 1}, t_{n+1})$:

$$T_i(x_{k\pm1}, t_{n+1}) = .5\bar{R}_i(x_{k\pm1}, 0)\bar{I}_i(x_{k\pm1}, n\Delta t)\,\Delta t + TT_i(x_{k\pm1}, t_{n+1}) + .5\bar{R}_i(x_{k\pm1}, n\Delta t)\bar{I}_i(x_{k\pm1}, 0)\,\Delta t \tag{21}$$

Relation (19) is important because it reduces CPU time greatly.

In all the above expressions, $\bar{R}_i(x_k, j\Delta t)$ is obtained from the IFFT of $\bar{R}_i(x, w)$. In the IFFT, the frequency range considered lies between $-\pi/\Delta t$ and $\pi/\Delta t$, so the time step is Δt too, which is required for numerical calculation of convolution. Theoretically the considered frequency range should be infinitely wide, but for any given driving signal the spectrum of response signals is finite, and if the time step Δt is fair little, $2\pi/\Delta t$ should be large enough to cover it. The sample number of IFFT should be large than or equal to the number of time samples to be analyzed.

In this paper, resistances are considered to be proportional to the square root of angular frequency (w) due to skin effect, i.e., $\bar{R}_i(x, w)$ $(i = 1, 2, \cdots, N)$ are all proportional to $\sqrt{w}$. Note that this isn't also a restriction to this method. Arbitrary relations between the resistances and frequency can be dealt with in the same way.

Insert (19) and (21) into (16) and (17) respectively, get

$$CKA1_i\bar{V}_i(x_{k+1}, t_{n+1}) + CKA2_i\bar{I}_i(x_{k+1}, t_{n+1}) = CKA_i \tag{22}$$

$$CKB1_i\bar{V}_i(x_{k-1}, t_{n+1}) - CKB2_i\bar{I}_i(x_{k-1}, t_{n+1}) = CKB_i \qquad i = 1, 2, \cdots, N \tag{23}$$

where

$$CKA1_i = 1 + .5\Delta t\bar{G}_i(x_{k+1})/\bar{C}_i(x_{k+1})$$

$$CKB1_i = 1 + .5\Delta t\bar{G}_i(x_{k-1})/\bar{C}_i(x_{k-1})$$

$$CKA2_i = \sqrt{\bar{L}_i(x_{k+1})/\bar{C}_i(x_{k+1})} + .5(\Delta t)^2\bar{R}_i(X_{k+1}, 0)/\sqrt{\bar{L}_i(x_{k+1})\bar{C}_i(x_{k+1})} - .5\Delta tFD_i(x_{k+1})$$

$$CKB2_i = \sqrt{\bar{L}_i(x_{k-1})/\bar{C}_i(x_{k-1})} + .5(\Delta t)^2\bar{R}_i(x_{k-1}, 0)/\sqrt{\bar{L}_i(x_{k-1})\bar{C}_i(x_{k-1})} - .5\Delta tFD_i(x_{k-1})$$

$$CKA_i = (1 - .5\Delta t\bar{G}_i(x_k)/\bar{C}_i(x_k))\bar{V}_i(x_k, t_n) + \sqrt{\bar{L}_i(x_k)/\bar{C}_i(x_k)}\bar{I}_i(x_k, t_n) - .5\Delta tT_i(x_k, t_n)/\sqrt{\bar{L}_i(x_k)\bar{C}_i(x_k)} - .5\Delta tTT_i(x_{k+1}, t_{n+1})/\sqrt{\bar{L}_i(x_{k+1})\bar{C}_i(x_{k+1})} + .5\Delta tFD_i(x_k)$$

$$CKB_i = (1 - .5\Delta t\bar{G}_i(x_k)/\bar{C}_i(X_k))\bar{V}_i(x_k, t_n) - \sqrt{\bar{L}_i(x_k)/\bar{C}_i(x_k)}\bar{I}_i(x_k, t_n) + .5\Delta tT_i(x_k, t_n)/\sqrt{\bar{L}_i(x_k)\bar{C}_i(x_k)} + .5\Delta tTT_i(x_{k-1}, t_{n+1})/\sqrt{\bar{L}_i(x_{k-1})\bar{C}_i(x_{k-1})} + .5\Delta tFD_i(x_k)$$

Equations (22) and (23) give the relations between transformed voltages and currents at closest segmenting points on the two set characteristic lines. If the transformed voltages and currents at every segmenting point on every single line at the time t_n is known, and boundary conditions are given, then the transformed voltages and currents at any segmenting point at time t_{n+1} can be obtained. This will be discussed in detail in next section.

III. Calculation Procedure

If boundary and initial conditions are known, using (22) and (23) the time response for given driving voltage can be obtained marching-on-in-time.

Fig. 1 gives the two set characteristic lines $dx/dt = 1/\sqrt{\bar{L}_i(x)\bar{C}_i(x)}$ (α lines on the figure) and $dx/dt = -1/\sqrt{\bar{L}_i(x)\bar{C}_i(x)}$ (β lines on the figure). From figure it is clear that the transformed voltage and current at the input end ($x = 0$) of the ith single line at time t_{n+1} can be obtained from the ones at the closest segmenting point (point A on figure) on the β line passing point $(0, t_{n+1})$. The coordinates of point A are (x_2, t_n). Applying (23), get

$$CKB1_i\bar{V}_i(0, t_{n+1}) - CKB2_i\bar{I}_i(0, t_{n+1}) = CKB_i \qquad i = 1, 2, \cdots, N. \tag{24}$$

Note that x_k in the expression of CKB_i is substituted by x_2 here.

Equation (24) can be written in matrix form:

$$[CKB1]\,[\bar{V}] - [CKB2]\,[\bar{I}] = [CKB] \tag{25}$$

where $[CKB1]$ and $[CKB2]$ are N-by-N diagonal matrices, $[\bar{V}]$, $[\bar{I}]$ and $[CKB]$ are column vectors.

The boundary condition at the input end ($x = 0$) of the transmission line system can be generally written as

$$[V] = f_1([I]) + [E]. \tag{26}$$

In which $[E]$ is the driving voltage column vector, $f_1(\cdot)$ indicates a function. Insert (11) and (12) into (26), one can get

$$[M]\,[\bar{V}] = f_1([M]\,[\bar{I}]) + [E]. \tag{27}$$

Combining (25) and (27), $[\bar{V}]$ and $[\bar{I}]$ at $x = 0$ and $t = t_{n+1}$ are gotten.

Similarly, at the output end ($x = D$), from the closest segmenting point on the α line passing (D, t_{n+1}), one can get

$$[CKA1]\,[\bar{V}] + [CKA2]\,[\bar{I}] = [CKA] \tag{28}$$

and the boundary condition at $x = D$ is

$$[M]\,[\bar{V}] = f_2([M]\,[\bar{I}]). \tag{29}$$

Combining (28) and (29), $[\bar{V}(D, t_{n+1})]$ and $[\bar{I}(D, t_{n+1})]$ are gotten.

At the segmenting points between $x = 0$ and $x = D$, such as point $H(x_k, t_{n+1})$ on Fig. 1, the transformed voltages and currents at time t_{n+1} are calculated from the closest segmenting points (such as point $G(x_{k-1}, t_n)$ and $I(x_{k+1}, t_n)$ on figure) on both α line and β line passing

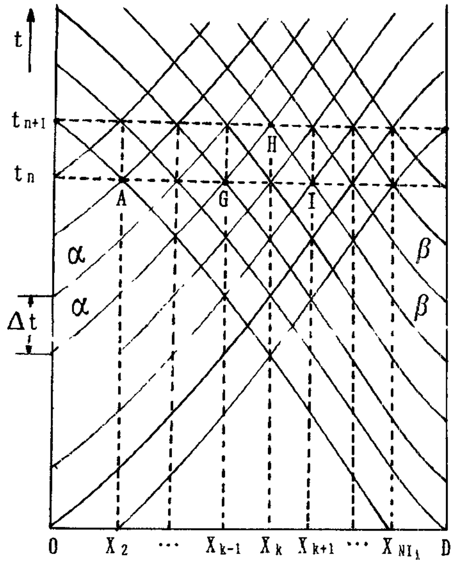

Fig. 1. Curves of characteristics.

these points, thus (22) and (23) both apply. For point H on Fig. 1, there is

$$[A]_{2\times 2}\begin{bmatrix} V_i(x_k, t_{n+1}) \\ I_i(x_k, t_{n+1}) \end{bmatrix} = [B] \qquad i = 1, 2, \cdots, N \tag{30}$$

where

$$[A] = \begin{bmatrix} CKA1_i & CKA2_i \\ CKB1_i & -CKB2_i \end{bmatrix}, \quad [B] = \begin{bmatrix} CKA_i \\ CKB_i \end{bmatrix}$$

$$i = 1, 2, \cdots, N.$$

Note that x_k in the expression of CKA_i is substituted by x_{k-1}, and x_k in the expression of CKB_i is substituted by x_{k+1}.

Now, the transformed voltages and currents at every segmenting point of every single line at time t_{n+1} are obtained; these values combined with the values before time t_{n+1} are then used to calculate the transformed voltages and currents at time t_{n+2}. If initial conditions are known, values at every time sample point can be obtained marching-on-in-time. At last, using (11) and (12), the time response at both input and output ends are gotten:

$$[V] = [M][\bar{V}] \qquad (\text{at } x = 0 \text{ or } x = D)$$

$$[I] = [M][\bar{I}] \qquad (\text{at } x = 0 \text{ or } x = D).$$

The above discussed procedure is for the general case of multiconductor transmission line system: nonuniform, having frequency-dependent losses and arbitrary terminals. For some special simple lines, the procedure can be simplified relevantly. For example, for uniform lines, the calculation of positions of every segmenting point is unnecessary, and for lines having frequency-independent parameters, the convolution calculation is unnecessary. Next section gives several examples for application of this method to various kinds of transmission lines.

IV. Examples and Discussions

Example 1: Two linearly loaded uniform lines (see Fig. 2).

The line parameters of the transmission system at 1 MHz in this example are

$$[L] = \begin{bmatrix} 309 & 21.7 \\ 21.7 & 309 \end{bmatrix} \text{nH/m}$$

$$[C] = \begin{bmatrix} 144 & -6.4 \\ -6.4 & 144 \end{bmatrix} \text{pF/m}$$

$$[R] = \begin{bmatrix} 8240 & 539 \\ 539 & 8240 \end{bmatrix} \text{m}\Omega/\text{m}$$

$$[G] = \begin{bmatrix} 905 & -11.8 \\ -11.8 & 905 \end{bmatrix} \text{nS/m}.$$

The resistances are assumed to vary proportionally to the square root of frequency. At one line end, one conductor is driven by a voltage generator $E(t)$ (in V):

$$E(t) = \begin{cases} 2t & t < .5 \text{ ns} \\ 1 & .5 \text{ ns} \le t < 5.5 \text{ ns} \\ 1 - 2(t - 5.5) & 5.5 \text{ ns} \le t < 6 \text{ ns} \\ 0 & t \ge 6 \text{ ns}. \end{cases}$$

The line length is $D = .3$ m. The boundary conditions at the input and output ends are

$$[V] = [I][R_1] + [E]$$

$$[V] = [R_2][I]$$

where

$$[R_1] = \begin{bmatrix} 50 & 0 \\ 0 & 75 \end{bmatrix} \Omega, \quad [R_2] = \begin{bmatrix} 50 & 0 \\ 0 & 50 \end{bmatrix} \Omega$$

$$[E] = \begin{bmatrix} E(t) \\ 0 \end{bmatrix}.$$

The results obtained with this method and with the frequency-domain method [6], [7] are both shown in Fig. 2. They are consistent as shown in the figure. For describing how the frequency-dependence of resistances affects the time response, a response voltage of the same transmission system, except the resistances of which are independent of frequency, is given in Fig. 2, also. It's easy to see that there are two main differences between these two cases. One is that the magnitudes of the response signals are different. Fig. 2 shows the magnitude of voltage at the output end of the driving line in the frequency-dependent case is lower than that in the frequency-independent case. It is understood that the frequency-dependence of resistances enlarges the line resistances, thus reduces the magnitude of response voltage at the output end of the driving line. Another is that the response signals in the frequency-dependent case are smoother, in which it is understood

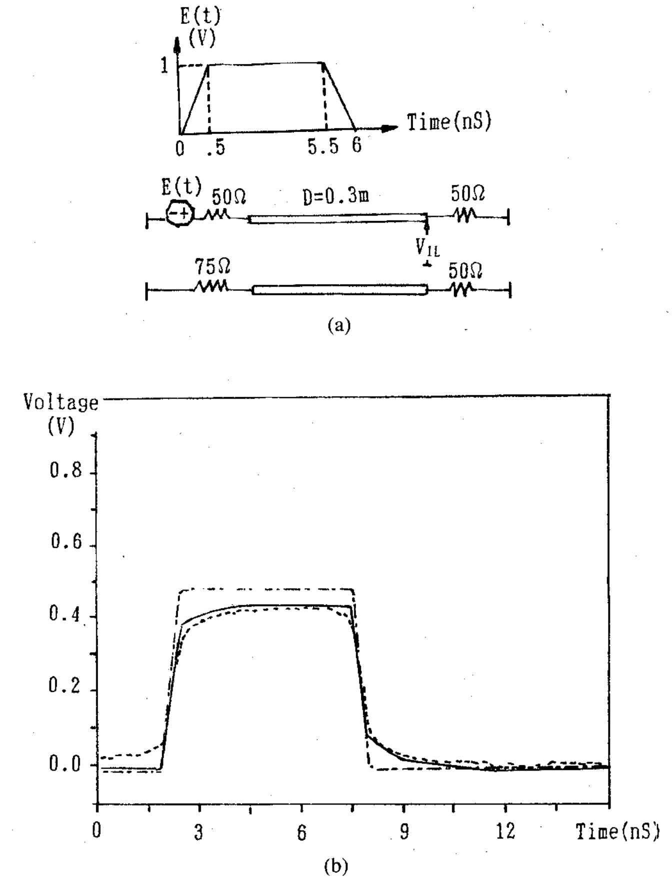

Fig. 2. (a) Transmission system in example one. (b) Response voltages at the output end of the driving line; —— with this method; ...with the method in [6], [7]; - · -the frequency-independent case.

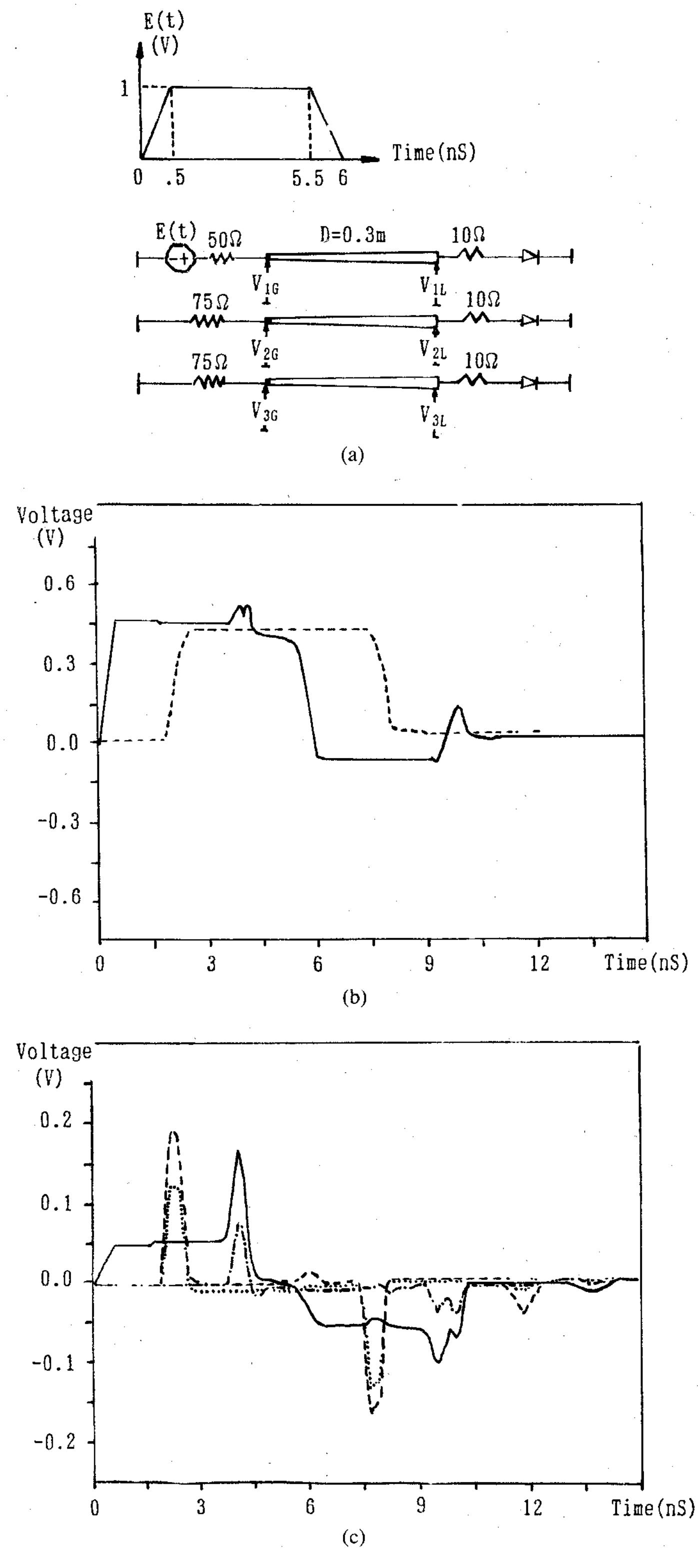

Fig. 3. (a) Transmission system in example two. (b) Response voltages on the driving line; —— at input end (V_{1G}); · · · at output end (V_{1L}). (c) Crosstalk voltages on the quiescent lines: ——V_{2G}; ---V_{2L}; - · -V_{3G}; · · ·V_{3L}.

that the frequency-dependence of resistances softens the response at high frequencies.

Example 2: Three nonlinearly loaded nonuniform lines (see Fig. 3).

The transmission system in this example is a general kind of line. The line circuit parameters are as follows:

$$l(x) = l_o/(1 + k_1(x))$$

$$l_m(x) = k_1(x)\, l(x)$$

$$k_1(x) = 0.1(1 + .6 \sin(\pi x + \pi/4))$$

$$c(x) = c_o/(1 - k_2(x))$$

$$c_m(x) = -k_2(x)\, c(x)$$

$$k_2(x) = 0.15(1 + .6 \sin(\pi x + \pi/4))$$

$$l_o = 387 \text{ nH/m}$$

$$c_o = 104.13 \text{ pF/m}$$

$$r = 1.2\ \Omega/\text{m at 1 MHz and is proportional to } \sqrt{|w|/2\pi}$$

$$r_m = 0$$

$$g = g_m = 0$$

where $l(x)$, $l_m(x)$ are the diagonal and second diagonal elements of the inductance matrix respectively, and the same meanings for $c(x)$, $c_m(x)$, $r(x)$, $r_m(x)$, $g(x)$ and $g_m(x)$. The driving voltage is shown in Fig. 3(a). The nonlinear loads are characterized by the relation

$$I = 10(e^{40V} - 1)$$

where I is the current in nA flowing through the loads, V is the corresponding voltage drop in volts. The results are shown in Fig. 3(b) and (c). The system of simultaneous nonlinear equations occurring in these examples are solved by the Newton Method.

In both of the above examples, convolution proves to be the main place where calculation time is consumed. Our calculations are performed on a PC-386/20. The numbers of the time samples and CPU time of these two examples are 350, 300, 390s, 720 s, respectively, thus this method is little time-consuming and convenient enough to be performed on MIC-PC.

On the other hand, in this method only the IFFT for $\bar{R}_i(x, w)$ is made, and the calculations proceed directly in the time-domain, i.e., FFT for the driving voltage and the relevant IFFT are unnecessary. As well known, direct FFT and IFFT for signals are the major factors inducing errors and oscillations in procedure of obtaining time response of multiconductor transmission lines. So, with this method the results are very stable with little oscillations and accurate even when the number of time samples is fair little, which can be seen from comparison of results obtained here with the ones obtained with other methods.

This method has still a advantage that it is straightforward and has some physical meanings. The crux lies in the convolution. The convolution occurring in this method has three advantages: first, it relates the voltages and currents at one time point to the ones before that time point and demonstrates clearly the cause and effect of signals; secondly, it includes the frequency-dependency of parameters in the time domain thus makes this method powerful; thirdly, so far, if considering the resistance to be frequency-dependent, then definition of resistance is given only in the frequency-domain, but using the convolution occurring in this method, the definition of resistance in the time domain can be given as below.

Considering a one-line transmission system. The telegrapher's equations in the time-domain and frequency-domain are

$$\partial V(x, t)/\partial x = -L(x)\,\partial I(x, t)/\partial t - R(x, t)I(x, t) \quad (31)$$

$$\partial V(x, w)/\partial w = -jwL(x)I(x, w) - R(x, w)I(x, w) \quad (32)$$

Using the inversely Fourier transform of (32), we get

$$\partial V(x, t)/\partial x = -L(x)\,\partial I(x, t)/\partial t - T(x, t) \quad (33)$$

where

$$T(x, t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} R(x, w)I(x, w)e^{jwt}\,dw$$

is just the convolution which can be calculated numerically as discussed before. Comparing (31) and (33), we get

$$R(x, t) = T(x, t)/I(x, t), \quad (34)$$

which is the definition of resistance in the time-domain. It indicates that resistance at any time is decided by both the system structure and the driving signal.

The main factor introducing errors in this method may be the fact that each decoupled single line couldn't just be segmented into integer number of parts in the characteristic method because of the different velocities of each transmission mode of multiconductor lines, thus suitable rounding off must be taken.

V. Conclusion

An effective new method for analysis of the time response of multiconductor transmission lines is presented. This method combines the functions of both convolution and characteristics, so the lines which can be analyzed may be nonuniform and arbitrarily loaded, and may have frequency-dependent losses. The advantage of this method is that it is powerful enough to analyze a wide range of transmission systems, some of which are not easy to analyze with other existing methods. It includes the frequency-dependence of line losses in the time-domain telegrapher's equations for the first time through the introduction of convolution. This kind of convolution may have other applications; for example, the definition of resistance can be given in the time-domain with it.

The crux of this method lies in inversely Fourier transforming the telegrapher's equations in the frequency-domain and obtaining a set of time-domain equations which can be numerically solved by the characteristic method. Because this method is in the time-domain, arbitrary terminals can be dealt with, and because of segmenting the lines in the characteristic method, nonuniform lines can be studied.

Two examples demonstrate that this method is correct and reliable, with stable and accurate results. This method is fast and convenient, so it can be performed on microcomputers of the PC series.

In our future work, the time response of nonuniform lines with frequency-dependent inductances and capacitances may be analyzed.

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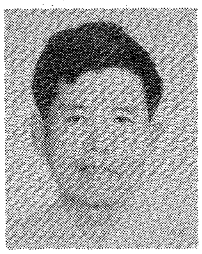

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